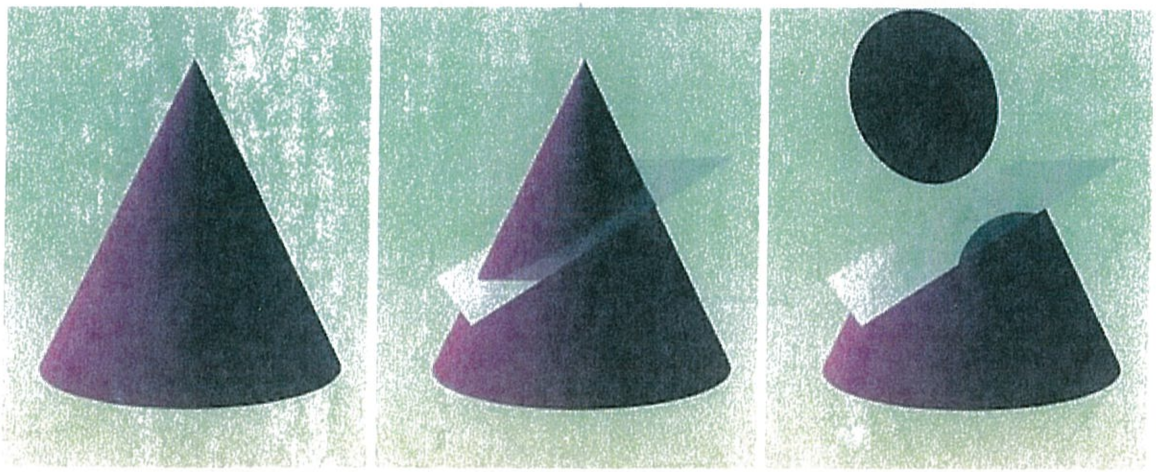
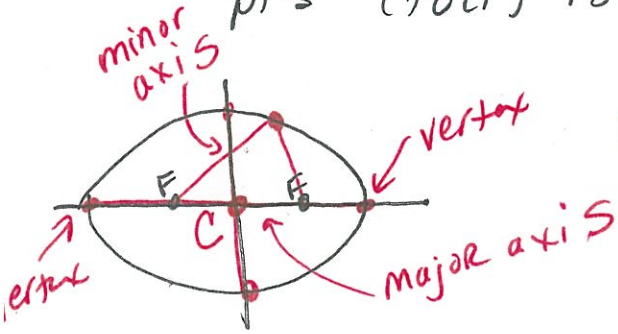
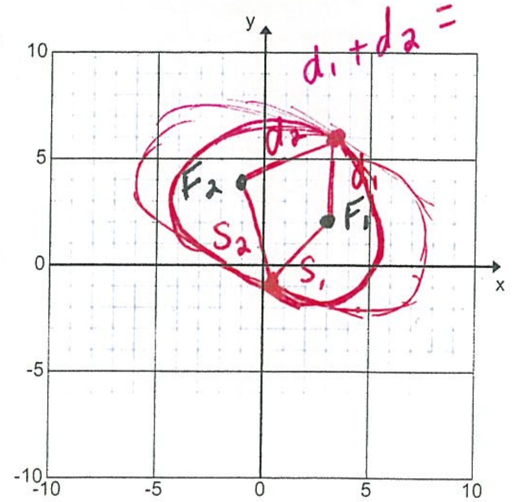
Conics Sections: Ellipse



Ellipse: set of pts. whose sum of 2 distances from 2 fixed pts (foci) is a constant.



Foci = Major axis.



Standard Forms: General FORM: $ax^2 + by^2 +$ — Both same sign. $A \neq B$

$a > b$ $\quad \dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$ Horizontal Ellipse

$\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$ Vert. Ellipse.

* a^2 is larger $\quad (a > b)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-h)^2}{b^2} = 1$$

Horizontal Ellipse:

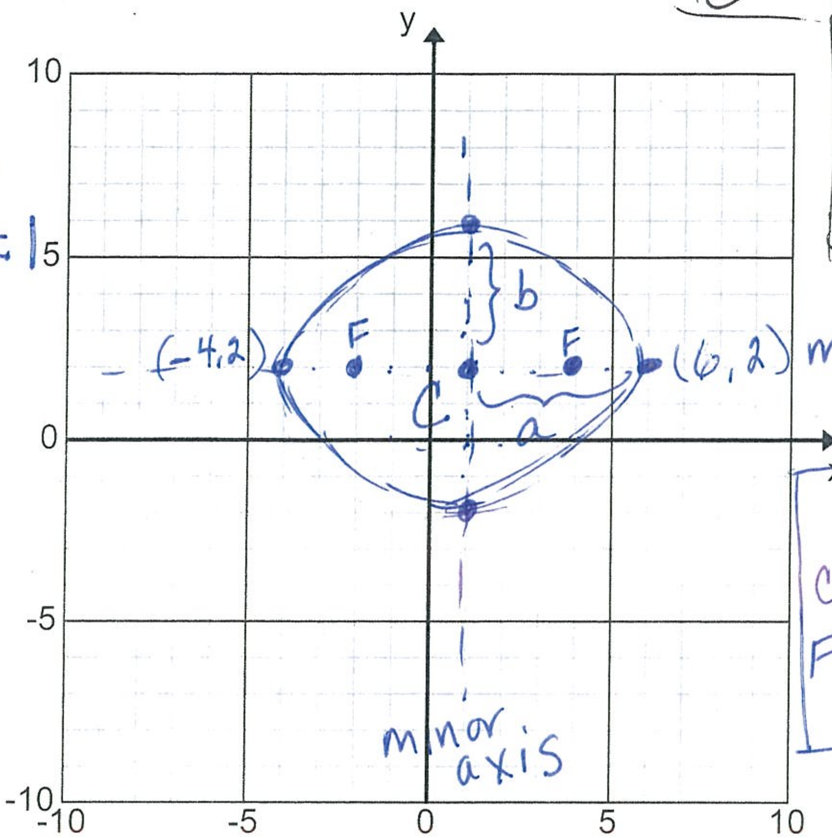
$\star$ $c^2 = a^2 - b^2$

a = vertices
b = co-vertices
c = foci

Ex:

$$\frac{(x-1)^2}{25} + \frac{(y-2)^2}{16} = 1$$

C: $(1,2)$
$a = 5$
$b = 4$ $\quad 25-16$
$c = 3$ $\quad \sqrt{9}$



V: $(6,2)$ & $(-4,2)$
CV: $(1,6)$ & $(1,-2)$
Foci: $(4,2)$ & $(-2,2)$

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$$\frac{(x-h)^2}{b^2} + \frac{(y-h)^2}{a^2} = 1$$

$\star$ eccentricity:
$e = c/a$
determines shape.

e closer to 0 circular

e closer to 1 elliptical

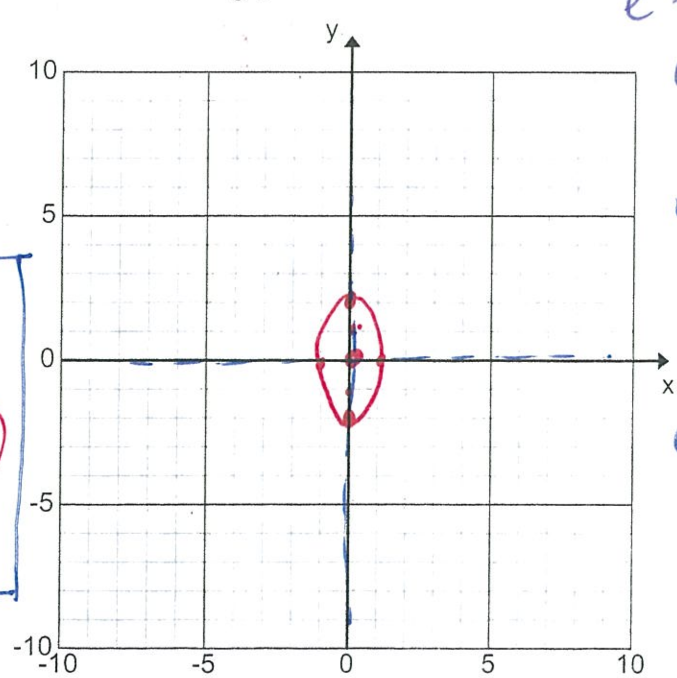
Vertical Ellipse:

Ex: $16x^2 + 4y^2 = 16$

$$\frac{x^2}{1} + \frac{y^2}{4} = 1$$

$a = 2$
$b = 1$
$c = 1$
C: $(0,0)$

V: $(0,2)$ $(0,-2)$
CV: $(1,0)$ $(-1,0)$
F: $(0,1)$ $(0,-1)$
$e = \frac{1}{2}$



Ex. $6x^2 + 2y^2 + 18x - 10y + 2 = 0$

$$6\left(x^2 + 3x + \tfrac{9}{4}\right) + 2\left(y^2 - 5y + \tfrac{25}{4}\right) = -2 + \tfrac{27}{2} + \tfrac{25}{2}$$

$$\frac{6\left(x + \tfrac{3}{2}\right)^2}{24} + \frac{2\left(y - \tfrac{5}{2}\right)^2}{24} = \frac{24}{24}$$

$$\frac{\left(x + \tfrac{3}{2}\right)^2}{4} + \frac{\left(y - \tfrac{5}{2}\right)^2}{12} = 1$$

$b^2 = 4, \quad b = 2$

$a^2 = 12, \quad a = 2\sqrt{3} \approx 3.5$

$c^2 = 8, \quad c = 2\sqrt{2} \approx 2.8$

C: $\left(-\tfrac{3}{2}, \tfrac{5}{2}\right)$

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V: $\left(-\tfrac{3}{2}, \tfrac{5}{2} \pm 2\sqrt{3}\right)$

CV: $\left(-\tfrac{3}{2} \pm 2, \tfrac{5}{2}\right)$

$\left(\tfrac{1}{2}, \tfrac{5}{2}\right) \quad \left(-\tfrac{7}{2}, \tfrac{5}{2}\right)$

F: $\left(-\tfrac{3}{2}, \tfrac{5}{2} \pm 2\sqrt{2}\right)$

$e = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{6}}{3} \approx .82$